INSTRUCTIONS TO CANDIDATES:

There are six questions. Attempt any four questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

NB: Extra information is supplied with this paper: Formulae sheet
Q1 (a) An element from a structure has a stress matrix in MPa of:

\[
\begin{pmatrix}
10 & 18 & 5 \\
18 & 15 & -10 \\
5 & -10 & 20 \\
\end{pmatrix}
\]

Draw the stress element identifying all of the stresses and their directions. (2 marks)

(b) If an element from a steel support structure has normal stresses of 125 MPa and 65 MPa in tension the x and z directions respectively. A compressive stress of 50 MPa acts also in the y direction. In addition an accompanying shear stress of 35 MPa in xy and -45 MPa in yz acts on the structure at this point; show using the stress tensor and that the principal stresses are approximately 133, 78 and -71 MPa respectively. (10 marks)

(c) Calculate the direction of maximum principal stress found in (b) in terms of angles relative to the xyz coordinate system. (5 marks)

(d) Show by the aid of a sketch the direction of this stress relative to the xyz coordinate system. (2 marks)

(e) If the yield strength of the material in tension and is 350 MPa and using a factor of safety of 2, determine if the factor of safety based upon the von Mises yield criterion and comment upon your findings. (5 marks)

Total 25 marks
Q2

(i) A 250mm internal diameter pipeline used in a chemical plant is pressurised to 2 MPa and has a blanking cover as shown schematically in fig Q2. If it is assumed that the cover is manufactured from steel with E=210 GPa and ν=0.31 and can be modelled as a thin flat circular plate. Hence show that

\[
\frac{\delta w}{\delta r} = \frac{pr^3}{16} + C_1 r^2 + C_2 \frac{1}{r} \frac{D}{D}
\]

Where: \( C_1 \) and \( C_2 \) are constants, \( p \) is the internal pressure, \( r \) is the radial distance from the centre of the plate, \( w \) is the normal displacement at position \( r \) and \( D = \frac{E}{12(1-\nu^2)} t^3 \)

(ii) Determine the necessary thickness \( t \) of the cover assuming that the design stress is limited to 205 MPa.

(iii) Does the thickness calculated justify the assumption? State the reason for your answer.

(iv) If in the actual cover it is secured by a ring of bolts, describe how this will change the stress value and the deflected shape under the load.
Q3

a) Boiler Plate steel is to be used to fabricate a cylindrical pressure vessel with a diameter \((2r)\) of 2m and a wall thickness \((t)\) of 8 mm with a fracture toughness of 43 MPa m\(^{1/2}\). Inspection reveals a crack of 25 mm length running in the circumferential direction.

(i) What is the maximum internal pressure \((P)\) allowable, assuming a safety factor of 3? Given the Hoop stress is \(Pr/t\) and longitudinal stress is \(Pr/2t\).

(ii) Consider how the allowable maximum internal pressure would be affected if the inspection showed the crack running in the axial direction.

(iii) If the internal pressure varies between 0.5 MPa to 1.5 MPa every 2 minutes, how many cycles are required to extend the crack to 50 mm in the circumferential direction and how long will this take? Assume \(C=10^{-36}\), \(Y=1\) and \(m = 4\)

b) (i) Explain the relationship between the stress intensity factor \(K\) and the fracture toughness \(K_c\) of a material.

(ii) Name two factors that \(K\) is dependent upon.

Total 25 Marks
Q4
(a) A 50 mm wide pultruded section fabricated from glass reinforced polyester is shown in fig Q4. The section is used as a cantilever 2.5m in length. Two beams are to be used to support a mass of 300 Kg. If the beam is designed to have a maximum design strain of 0.2%, determine the factor of safety for the beams. (17 Marks)

(b) Sketch the stress distribution through the depth of each beam and indicate the salient values. (5 Marks)

(c) Comment also on the efficiency of the design. (3 Marks)

Assume for the materials used the following values:

<table>
<thead>
<tr>
<th>Material</th>
<th>Key</th>
<th>Efficiency Factor</th>
<th>Modulus (GPa)</th>
<th>Volume Fraction (%)</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uni-Directional</td>
<td></td>
<td>0.9</td>
<td>68</td>
<td>60</td>
<td>12mm</td>
</tr>
<tr>
<td>Woven Roving</td>
<td></td>
<td>0.45</td>
<td>68</td>
<td>45</td>
<td>10mm</td>
</tr>
<tr>
<td>CSM</td>
<td></td>
<td>0.25</td>
<td>68</td>
<td>30</td>
<td>10mm</td>
</tr>
<tr>
<td>Polyester Resin</td>
<td></td>
<td>-</td>
<td>3</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig Q4
Q5

(a) A high performance motorsports stiffening beam is fabricated from a carbon fibre reinforced skin and an aluminium honeycomb core. If the beam is limited to a width of 90 mm and has the loading shown in fig Q5. Determine for this arrangement a suitable skin thickness for the beam. (16 Marks)

(b) If the beam has to withstand an extra torsional load describe how the beam layup would change. (5 Marks)

(c) If the beam is subjected to cyclic temperature loading describe what problems could occur with the aluminium honeycomb. (4 marks)

Assume the following properties of the UD carbon fibre prepreg:
\( E_1 = 310 \) GPa, \( E_2 = 9 \) GPa, Volume Fraction 65%, design strain 0.6% interface adhesive strength 12 MPa and prepreg thickness is 0.125mm/layer.

125 KN/m UDL
A tubular section, 80mm NB x 3mm wall thickness is used to fabricate part of an anti-roll beam. The beam can be assumed to be rigid connected on the left side (A) and pinned at points B and C with no temperature change. The setup and loading is shown schematically in Fig Q6.

(a) If you assume the material behaves rigid perfectly plastic with a yield stress of 350 MPa calculate the factor of safety for the system. (16 marks)

(b) If the right hand support (C) is assumed to be fixed determine the new factor of safety. (5 marks)

(c) What would be the increase in load carrying capacity if the wall thickness of the tube is increased by 50%? (4 marks)
Fig Q6 Schematic layout

FORMULA SHEET

Formulae used in Structures and Materials Module

Elasticity – finding the direction vectors

\[
\begin{bmatrix}
S_x \\
S_y \\
S_z
\end{bmatrix} = (\text{Stress Tensor}) \begin{bmatrix}
l \\
m \\
n
\end{bmatrix}
\]

\[
k = \frac{1}{\sqrt{a^2 + b^2 + c^2}}
\]
University of Bolton
BEng (Hons) Mechanical Engineering
Semester 1 2015/2016
Advanced Materials and Structures
Module No. AME6002

Where a, b and c are the co-factors of the eigenvalue stress tensor.

\[ l = ak \quad l = \cos \alpha, \]
\[ m = bk \quad m = \cos \theta, \]
\[ n = ck \quad n = \cos \varphi. \]

**Principal stresses and Mohr’s Circle**

\[ \tau_{12} = \frac{\sigma_1 - \sigma_2}{2} \]
\[ \tau_{13} = \frac{\sigma_1 - \sigma_3}{2} \]
\[ \tau_{23} = \frac{\sigma_2 - \sigma_3}{2} \]

**Yield Criterion**

**Von Mises**

\[ \sigma_{\text{von Mises}} = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} \]

**Tresca**

\[ \sigma_3 \geq \sigma_2 \geq \sigma_1 \]
\[ \sigma_{\text{Tresca}} = 2 \cdot \tau_{\text{max}} \]
\[ \tau_{\text{max}} = \max \left( \frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_1 - \sigma_3}{2}, \frac{\sigma_3 - \sigma_2}{2} \right) \]
\[ \frac{\sigma_{\text{von Mises}}}{\sigma_{\text{Tresca}}} = \frac{\sqrt{3}}{2} \]
shear stress varies inversely with thickness \( \tau = \frac{T}{2ta} \)

shear flow \( q \)

shear flow = \( q = \tau t \)

Applied torque \( T = 2qA \)

Angle of twist

\[
\phi = \frac{TL}{4A^2G} \oint ds \frac{q}{t}
\]

Torsion in multi-cells thin wall cross section

Section considered as an assembly of \( N \) tubular sub-sections (compartments), each subjected to torque \( T_i \) as shown on the figure below:

Total torque

\[
T = \sum_{i=1}^{N} T_i = 2 \cdot \sum_{i=1}^{N} q_i A_i
\]

Common angle of twist for all compartments:

\[
\theta = \frac{L}{2GAt} \oint \frac{q_i - q'}{t(s)} ds
\]

Where \( q \) is the shear flow of the main compartment, \( q' \) is the shear flow due to torque in adjacent compartments, \( A_i \) the area of cross-section \( i \), \( t \) is the thickness of the cross-section and \( s \) is the circumference of the compartment.
Torsion in open thin wall cross section (OTW)

If \( \frac{b}{t} \geq 10 \) then \( \alpha = \beta = \frac{1}{3} \)

and \( J_{\alpha} = J_{\beta} = J = \sum_{i=1}^{n} \frac{1}{3} b_i t_i^3 \)

Shear stress
\( \tau_{\text{max}} = \frac{T t_{\text{max}}}{J} \)

Twist angle
\( \phi = \frac{LT}{GJ} \)

(a) \( J = \frac{1}{3} bt^3 \)

(b) \( J = \frac{1}{3} b_1 t_1^3 + \frac{1}{3} b_2 t_2^3 \)

(c) \( J = \frac{1}{3} b_1 t_1^3 + \frac{1}{3} b_2 t_2^3 + \frac{1}{3} b_3 t_3^3 \)

* b: long dimension
  * t: thickness
**Fracture mechanics**

### Table: $Y$ values for plates loaded in tension

<table>
<thead>
<tr>
<th>Condition</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Through crack of length $2a$ in an infinite plate</td>
<td>$Y = \left( \frac{\pi a}{w} \right)^{1/2} \frac{2a}{w} \leq 0.7$</td>
</tr>
<tr>
<td>Edge crack of length $a$ in an infinite plate</td>
<td>$Y = 1.12$</td>
</tr>
<tr>
<td>Because plane strain and plane stress have identical stress fields, this calibration is also for an edge scratch of depth $a$ on a large body carrying tensile stress $\sigma$.</td>
<td></td>
</tr>
<tr>
<td>Through crack of length $2a$ in a plate of width $w$.</td>
<td>$Y = 0.265 \left( \frac{b}{w} \right)^4 + \frac{0.875 + 0.265a/w}{(b/w)^{1/2}}$</td>
</tr>
<tr>
<td>Edge crack of length $a$ in a plate of width $w$.</td>
<td>$Y = 0.265 \left( \frac{b}{w} \right)^4 + \frac{0.875 + 0.265a/w}{(b/w)^{1/2}}$</td>
</tr>
</tbody>
</table>
Life Calculations

$$\frac{da}{dN} = C (\Delta K)^m$$

$$N = \frac{1}{CY^m \sigma^m \pi^2 a_0 a^2} \int_{a}^{\alpha_i} \frac{da}{C Y^m \sigma^m \pi^2 a_0 a^2}$$

(5) Penny-shaped internal crack of radius $a$.

$Y = \frac{2}{\pi}, \quad a \ll D$

(6) Semi-elliptical surface flaw

$Y = \frac{1.12}{\phi^{1/2}}$
Circular Plates

\[ \frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} \left( r \frac{dw}{dr} \right) \right] = -\frac{Q_r}{D} \]

Hooke’s law is expressed in terms of \( w \), as follows

\[
\sigma_r = \frac{E}{1-\nu^2} (\varepsilon_r + \nu \varepsilon_\theta) = -\frac{Ez}{1-\nu^2} \left( \frac{d^2 w}{dr^2} + \frac{v}{r} \frac{dw}{dr} \right)
\]
\[
\sigma_\theta = \frac{E}{1-\nu^2} (\varepsilon_\theta + \nu \varepsilon_r) = -\frac{Ez}{1-\nu^2} \left( \frac{1}{r} \frac{dw}{dr} + \nu \frac{d^2 w}{dr^2} \right)
\]

Bending moment and shear force

\[
M_r = -D \left( \frac{d^2 w}{dr^2} + \frac{v}{r} \frac{dw}{dr} \right), \quad D = \frac{Et^3}{12 (1-\nu^2)}
\]
\[
M_\theta = -D \left( \frac{1}{r} \frac{dw}{dr} + \nu \frac{d^2 w}{dr^2} \right)
\]
\[
Q_r = -\frac{1}{2\pi} \int_0^{2\pi} \int_b^{r} grdrd\theta = -\frac{1}{r} \int_b^{r} grdr
\]

Governing equation

\[
\nabla^4 w = \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) w = \frac{q}{D}
\]
### Plastic Sections

<table>
<thead>
<tr>
<th>Section</th>
<th>Elastic Modulus</th>
<th>Plastic Modulus</th>
<th>Shape Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>$\frac{1}{6}bh^2$</td>
<td>$\frac{1}{4}bh^2$</td>
<td>1.50</td>
</tr>
<tr>
<td>circle</td>
<td>$\frac{\pi}{32}D^3$</td>
<td>$\frac{1}{6}D^3$</td>
<td>1.70</td>
</tr>
<tr>
<td>thick-walled tube</td>
<td>$\frac{\pi}{32}D^3\left[1-\left(1-\frac{2t}{D}\right)^4\right]$</td>
<td>$\frac{1}{6}D^3\left[1-\left(1-\frac{2t}{D}\right)^3\right]$</td>
<td>± 1.50</td>
</tr>
<tr>
<td>thin-walled tube</td>
<td>$\frac{\pi}{4}tD^2$</td>
<td>$tD^2$</td>
<td>1.27</td>
</tr>
<tr>
<td>I-section</td>
<td>$bht + \frac{1}{6}dh^2$</td>
<td>$bht + \frac{1}{4}dh^2$</td>
<td>± 1.15</td>
</tr>
<tr>
<td>T-section</td>
<td>$\frac{5}{18}ta^2$</td>
<td>$\frac{1}{2}ta^2$</td>
<td>1.80</td>
</tr>
</tbody>
</table>

Position of the Maximum moment of a propped cantilever length L is given by:
Cubic Equations-General form

\[ \sigma_3 + F_1 \sigma_2 + F_2 \sigma + F_3 = 0 \]

where: \( F_1, F_2, \) & \( F_3 \) are constants then the solution has three roots, say \( a, b \) & \( c \), giving:

\[ (\sigma-a)(\sigma-b)(\sigma-c) = 0, \]

hence,

\[ \sigma^3 - \sigma^2(a+b+c) + \sigma(ab+bc+ca) - abc = 0 \]

as a general form.

If either \( a, b \) or \( c \) is known a simple quadratic equation based upon the other two unknowns can derived and solved.

Composite materials

\[ E_{\text{composite}} = E_{\text{fibre}}V_{\text{fibre}} + E_{\text{matrix}}(1 - V_{\text{fibre}}) \]

Fracture Toughness

### Table: Fracture toughness of some engineering materials

<table>
<thead>
<tr>
<th>Material</th>
<th>( K_{IC} ) (MNm(^{3/2}))</th>
<th>( E ) (GN/m(^2))</th>
<th>( \Sigma_{IC} ) (kJ/m(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain carbon steels</td>
<td>140 - 200</td>
<td>200</td>
<td>100 - 200</td>
</tr>
<tr>
<td>High strength steels</td>
<td>30 - 150</td>
<td>200</td>
<td>5 - 110</td>
</tr>
<tr>
<td>Low to medium strength steels</td>
<td>10 - 100</td>
<td>200</td>
<td>0.5 - 50</td>
</tr>
<tr>
<td>Titanium alloys</td>
<td>30 - 120</td>
<td>120</td>
<td>7 - 120</td>
</tr>
<tr>
<td>Aluminium alloys</td>
<td>22 - 33</td>
<td>70</td>
<td>7 - 16</td>
</tr>
<tr>
<td>Glass</td>
<td>0.3 - 0.6</td>
<td>70</td>
<td>0.002 - 0.008</td>
</tr>
<tr>
<td>Polycrystalline alumina</td>
<td>5</td>
<td>300</td>
<td>0.08</td>
</tr>
<tr>
<td>Teak – crack moves across the grain</td>
<td>8</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.4</td>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>PMMA (Perspex)</td>
<td>1.2</td>
<td>4</td>
<td>0.4</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>1.7</td>
<td>3</td>
<td>0.01</td>
</tr>
<tr>
<td>Polycarbonate (ductile)</td>
<td>1.1</td>
<td>0.02</td>
<td>54</td>
</tr>
<tr>
<td>Polycarbonate (brittle)</td>
<td>0.4</td>
<td>0.02</td>
<td>6.7</td>
</tr>
<tr>
<td>Epoxy resin</td>
<td>0.8</td>
<td>3</td>
<td>0.2</td>
</tr>
<tr>
<td>Fibreglass laminate</td>
<td>10</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>Aligned glass fibre composite – crack across fibres</td>
<td>10</td>
<td>35</td>
<td>3</td>
</tr>
<tr>
<td>Aligned glass fibre composite – crack down fibres</td>
<td>0.03</td>
<td>10</td>
<td>0.0001</td>
</tr>
<tr>
<td>Aligned carbon fibre composite – crack across fibres</td>
<td>20</td>
<td>185</td>
<td>2</td>
</tr>
</tbody>
</table>