# UNIVERSITY OF BOLTON

# SCHOOL OF THE BUILT ENVIRONMENT & ENGINEERING

# MECHANICAL ENGINEERING

# SEMESTER ONE EXAMINATION 2009/2010

# ENGINEERING ANALYSIS 2

# MODULE NO: MEC2004

# GERMAN STUDENT GROUP ONLY

**Date: Monday, 18 January 2010** **Time: 2.00 – 4.00 p.m.**

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**INSTRUCTIONS TO CANDIDATES:**

There are FIVE questions.

Answer ALL questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Unless otherwise stated all symbols take their usual meaning

Electronic calculators may be used provided the data and program storage memory is cleaned prior to the examination.

**CANDIDATES REQUIRE :** Formula Sheet (attached)

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School of the Built Environment & Engineering
Mechanical Engineering
Semester 1 Examination 2009/2010
Engineering Analysis 2
Module No. MEC2004
German Student Group Only

Q1 a) Show from first principles that for x = F(t) :

$$\mathcal{L}\left(\frac{dx}{dt}\right) = s\bar{x} - x_0$$ (5 marks)

and indicate the expected format for $\mathcal{L}\left(\frac{d^2x}{dt^2}\right)$, clearly defining what each term means. (2 marks)

b) For the second order differential equation of the format :

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx = F(t)$$

where a = -4, b = 4 and F(t) = 4cos2t and given that x = 2 and $\frac{dx}{dt}$ = 5 when t = 0, show using Laplace transforms that :

$$x = 2e^{2t}(1+t) - \tfrac{1}{2}\sin 2t$$

is the solution of the differential equation. (13 marks)

**Total 20 marks**

Q2 a) Briefly describe, with the aid of a diagram, the basic Euler method used for finding the solution of differential equations numerically.
(5 marks)

b) For the first order differential equation :

$$\frac{dy}{dx} = x + y$$

with the initial condition that y = 0 at x =0, use the basic Euler numerical method to obtain solutions of the differential equation over the range x=0 to x = 0.6 using step intervals of 0.1. (8 marks)

**Question 2 continued over**

**Question 2 continued**

c) If the actual solution of the differential equation is :

$$y = e^x - x - 1$$

verify that this is a valid equation and compare the exact value of y(0.6) with that obtained numerically. (5 marks)

d) Comment on how the accuracy of the numerical method might be improved. (2 marks)

**Total 20 marks**

Q3 a) Sketch and fully label the region enclosed by the functions $y = 2x$, $y = 6-x$ and the x axis. (4 marks)

b) For the region in a) above, use double integration to determine the volume over the region if the function defining the surface above the region is $z = x(x^2 - y^2)$.

NB : You must develop the double integration so that integration with respect to x occurs first. (10 marks)

c) For the region in a) specify clearly the required double integration needed to determine the volume under the surface described in b) but for the case when integration with respect of y occurs first.

NB : Do not evaluate the integration. (6 marks)

**Total 20 marks**

**Please turn the page**

School of the Built Environment & Engineering
Mechanical Engineering
Semester 1 Examination 2009/2010
Engineering Analysis 2
Module No. MEC2004
German Student Group Only

Q4 A function f(x), which is periodic in 2π may be defined by :

$$f(x) = -x \qquad -\pi \le x < 0$$

$$f(x) = 0 \qquad 0 \le x < \pi$$

a) Sketch the function and determine its Fourier coefficients. (15 marks)

b) Show that the function can be represented by the Fourier series :

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \ldots\right)$$
$$+\left(-\sin x + \frac{1}{2}\sin 2x - \frac{1}{3}\sin 3x + \frac{1}{4}\sin 4x - \ldots\right)$$

(5 marks)

**Total 20 marks**

Q5 a) The time of oscillation T of a simple pendulum is give by :

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Where $\ell$ is the length of the pendulum and g is the free fall acceleration due to gravity.

Determine $\frac{\partial T}{\partial \ell}$ and $\frac{\partial T}{\partial g}$ (12 marks)

b) The radius of a right circular cylinder is increasing at a rate of 2 cm/s and the height is decreasing at a rate of 3 cm/s.

Find the rate at which the volume is changing when the radius is 8 cm and the height is 5 cm. (8 marks)

**Total 20 marks**

**END OF QUESTIONS**

School of the Built Environment & Engineering
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Semester 1 Examination 2009/2010
Engineering Analysis 2
Module No. MEC2004
German Student Group Only

## FORMULA SHEET

Laplace

$$\mathcal{L}(x) = \int_0^\infty x.e^{-st}\ dt$$

| Function | Transform |
|---|---|
| $a$ | $\frac{a}{s}$ |
| $e^{at}$ | $\frac{1}{s-a}$ |
| $e^{-at}$ | $\frac{1}{s+a}$ |
| $t$ | $\frac{1}{s^2}$ |
| $\sin at$ | $\frac{a}{s^2+a^2}$ |
| $\cos at$ | $\frac{s}{s^2+a^2}$ |

1st Shift Theorem

$$\mathcal{L}(e^{-at}\,.\,F(t)) = f(s+a)$$

$$\mathcal{L}(e^{at}\,.\,F(t)) = f(s-a)$$

Partial Fractions

$$\frac{F(x)}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$$

$$\frac{F(x)}{(x+a)(x+b)^2} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+b)^2}$$

$$\frac{F(x)}{(x^2+a)} = \frac{Ax+B}{(x^2+a)}$$

School of the Built Environment & Engineering
Mechanical Engineering
Semester 1 Examination 2009/2010
Engineering Analysis 2
Module No. MEC2004
German Student Group Only

## Numerical

Euler : $y_1 \approx y_0 + h\, y_0^1$

Improved Euler : $y_1 \approx y_0 + \frac{1}{2}h(y_0^1 + f(x_1, \bar{y}_1))$

## Integration by Parts

$$\int u.\frac{dv}{dx}dx = uv - \int v.\frac{du}{dx}dx$$

## Fourier

$$F(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).\cos nx\, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).\sin nx\, dx$$

## Max/Min

$C = F(x,y)$

Stationary Points $\frac{\partial C}{\partial x} = 0, \quad \frac{\partial C}{\partial y} = 0$

$$\frac{\partial^2 C}{\partial x^2}.\frac{\partial^2 C}{\partial y^2} - \left(\frac{\partial^2 C}{\partial x \partial y}\right)^2$$

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Semester 1 Examination 2009/2010
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German Student Group Only

Small Changes

z = f(u, v, w)

$$\delta z \simeq \frac{\partial z}{\partial u}.\delta u + \frac{\partial z}{\partial v}.\delta v + \frac{\partial z}{\partial w}.\delta w$$

Total Differential

z = f(u, v, w)

$$dz = \frac{\partial z}{\partial u}du + \frac{\partial z}{\partial v}dv + \frac{\partial z}{\partial w}dw$$

Rate of Change

z = f(u, v, w)

$$\frac{dz}{dt} = \frac{\partial z}{\partial u}\cdot\frac{du}{dt} + \frac{\partial z}{\partial v}\cdot\frac{dv}{dt} + \frac{\partial z}{\partial w}\cdot\frac{dw}{dt}$$