

**UNIVERSITY OF BOLTON**

**SCHOOL OF THE BUILT ENVIRONMENT &  
ENGINEERING**

**FdSc/HNC CIVIL ENGINEERING**

**SEMESTER ONE EXAMINATION 2009/2010**

**MATHEMATICS B**

**MODULE NO: BLT 2120**

Date: Tuesday 19 January 2010

Time: 10.00 am – 12.00 noon

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**INSTRUCTIONS TO CANDIDATES:**

There are **FOUR** questions.

Candidates should either:

Answer **THREE FULL** questions or

Answer parts of **FOUR** questions which carry a total not exceeding 75 marks.

All questions carry a **Total** of 25 marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a maximum total of 75 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

A formula sheet is attached.

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1. a) Find the first and second derivatives of the following functions:

i)  $y = 2\sqrt[3]{x^5} + \frac{3}{x^2}$  (3 marks)

ii)  $y = 3e^{-4x} + 2 \cos 3x$  (3 marks)

b) Differentiate the following functions using the "function of a function rule":

i)  $y = \sqrt[3]{4x^2 + x - 1}$  (3 marks)

ii)  $y = \sin(3x + 2)$  (3 marks)

c) Differentiate the following functions to find  $dy/dx$ :

i)  $y = 5e^{2x} \sin 2x$  using the "product rule" (4 marks)

ii)  $y = \frac{x^3 - 1}{\ln x}$  using the "quotient rule" (4 marks)

d) Find the coordinates of the maximum and minimum points for the function  $y = 4x^3 - 33x^2 + 72x - 30$  (5 marks)

**Total 25 marks**

2. (a) Evaluate the following indefinite integrals using integration by parts:

(i)  $\int 6xe^{4x} dx$

(ii)  $\int 4x^3 \ln 2x dx$

(iii)  $\int 2x^2 \sin 3x dx$

(15 marks)

**Question 2 continued over the page...**

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**Question 2 continued**

(b) Estimate the following definite integral using:

- (i) the trapezoidal rule with four strips,
- (ii) Simpson's rule with four strips.

$$\int_{1/3}^1 (4x + 3x^2) dx$$

Calculate and comment on the percentage errors in your results.

(10 marks)

**Total 25 marks**

3. (a) For the matrices

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \\ 1 & -1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} -1 & 2 & 1 \\ -1 & 1 & 2 \\ 1 & -1 & 2 \end{pmatrix}$$

calculate the following:

- (i)  $A + B$       (ii)  $AB$       (iii)  $BA$

Comment on your answers to (ii) and (iii)

(10 marks)

(b) Solve the following system of linear simultaneous equations using matrix inversion:

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 9 \\ 4x_1 + 3x_2 - x_3 &= -1 \\ 3x_1 - 2x_2 + 2x_3 &= 9 \end{aligned}$$

(15 marks)

**Total 25 marks**

**Please turn the page**

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4. a) Evaluate the following definite integrals:

i)  $\int_3^4 (x^3 + 4x^2) dx$  (3 marks)

ii)  $\int_{-1}^2 \left( x^2 + \frac{2}{x^2} \right) dx$  (3 marks)

b) Integrate the following expressions using simple algebraic substitution:

i)  $\int (6x + 3)^4 dx$  (3 marks)

ii)  $\int 10x^3 (6x^4 + 3)^3 dx$  (3 marks)

c) Use Pascal's triangle to expand the following:

i)  $(x - 2)^3$  (3 marks)

ii)  $(3x + 5)^4$  (3 marks)

d) Expand each of the following using the binomial theorem:

i)  $(1 - x)^4$  (3 marks)

ii)  $(5 + 2x)^3$  (4 marks)

**Total 25 marks**

**END OF QUESTIONS**

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### Formula sheet

$y = f(x)$	$\frac{dy}{dx}$
Constant (k)	0
x	1
$x^2$	2x
$x^n$	$nx^{n-1}$
$e^x$	$e^x$
$e^{kx}$	$ke^{kx}$
sin x	cos x
cos x	- sin x
sin kx	k cos kx
cos kx	-k sin kx
tan kx	$k \cdot \sec^2 kx$
ln kx	$\frac{1}{x}$

$y = f(x)$	$\int y \, dx$ all '+c'
2x	$x^2$
x	$\frac{1}{2} x^2$
k (constant)	k x
$x^n$ ( $n \neq -1$ )	$\frac{1}{n+1} x^{n+1}$
$\frac{1}{x}$	ln x
$e^x$	$e^x$
$e^{kx}$	$\frac{e^{kx}}{k}$
sin x	- cos x
cos x	sin x
sin kx	$-\frac{\cos kx}{k}$
cos kx	$\frac{\sin kx}{k}$

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**Formula Sheet (continued)**

Coefficients in the Binomial expansion													
							1						
						1		1					
					1		2		1				
				1		3		3		1			
			1		4		6		4		1		
		1		5		10		10		5		1	
	1		6		15		20		15		6		1

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!} a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!} a^{n-3}b^3 + \dots + b^n$$

If  $y = u.v$  then  $\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

If  $y = \frac{u}{v}$  then  $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

If  $y$  is a function of  $u$ , and  $u$  is a function of  $x$  then  $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

**Integration by parts:**  $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

The inverse of a matrix  $A$  is  $A^{-1} = \left[ \frac{Adj A}{|A|} \right]$

where  $|A|$  is the determinant of  $A$  and  $Adj A$  is the transpose of the matrix of cofactors of  $A$ .

**Simpson's Rule:** Area =  $\frac{h}{3} \{(y_1 + y_n) + 4(y_2 + y_4 + \dots + y_{n-1}) + 2(y_3 + y_5 + \dots + y_{n-2})\}$

**Trapezium Rule:** Area =  $h \left\{ \frac{y_0 + y_n}{2} + (y_1 + y_2 + y_3 + \dots + y_{n-1}) \right\}$