UNIVERSITY OF BOLTON

DEPARTMENT OF THE BUILT ENVIRONMENT & ENGINEERING

MECHANICAL ENGINEERING

SEMESTER ONE EXAMINATION 2008/2009

ENGINEERING ANALYSIS 2

MODULE NO: MEC2004

GERMAN STUDENT GROUP ONLY

Date: Wednesday, 14 January 2009 **Time: 2.00 – 4.00 p.m.**

INSTRUCTIONS TO CANDIDATES:

There are FIVE questions.

Answer ALL questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Unless otherwise stated all symbols take their usual meaning

Electronic calculators may be used provided the data and program storage memory is cleaned prior to the examination.

CANDIDATES REQUIRE : **Formula Sheet (attached)**

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Q1 a) The Laplace transform of x where x = f(t) is defined as :

$$(x) = \int_0^\infty x.e^{-st} dt$$

Use this definition to show that :

$$\left(\frac{dx}{dt}\right) = s\bar{x} - x_0$$

where the symbols have their usual meaning. (8 marks)

b) Using Laplace transforms, solve the first order differential equation :

$$\frac{dx}{dt} - 3x = t.e^{2t}$$

given the initial condition that x = 0 when t = 0. (12 marks)

Total 20 marks

Q2 a) The climb of an aircraft soon after take-off and over a given distance may be considered to be related by the differential equation :

$$\frac{dy}{dx} = x + y$$

with the initial condition that at some datum distance from take-off x = 0, its height y = 1 unit, where 1 unit represents 100 metres.

Use the “improved” Euler method to obtain the solution of the differential equation over the range x = 0 to x = 0.5 km in steps of 0.1 km. (12 marks)

b) If the particular solution to the differential equation in a) is given as :

$y = 2e^x - x - 1$

verify that this solution satisfies the initial condition and the differential coefficient. (4 marks)

Question 2 continued over

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Question 2 continued

c) At x = 0.5 km, determine y(0.5) and compare the value to the corresponding numerical value indicating the % error. (4 marks)

Total 20 marks

Q3 A hardened and formed punch required for a punch and die operation on a crank forging is such that the effective volume of the punch that is defined over a given region is given by the double integral :

$$V = \int_0^1 \int_y^{\sqrt{2-y^2}} \frac{x}{\sqrt{x^2+y^2}}\,dxdy$$

a) Sketch and fully label the region that this integral describes. (4 marks)

b) Evaluate the volume. (8 marks)

c) Reverse the order of integration and develop the format of integration needed to evaluate the volume and then comment on why this format tends towards a more difficult mode of solution. (8 marks)

N.B. Do not attempt the solution here.

Total 20 marks

Q4 a) Comment on what you understand to be meant by the expression "Fourier Sine Series".

Indicate what types of function this series relates to and give an example. (5 marks)

b) For the function described by :

f(x) = -6	$-\pi < x < 0$
f(x) = 6	$0 < x < \pi$

Question 4 continued over

Question 4 continued

(i) Sketch and label the function. (3 marks)

(ii) Develop the Fourier series that represents the function.

(12 marks)

Total 20 marks

Q5 Thin sheet metal is to be used to make an open top container of rectangular section which is to have a volume of $10m^3$. The width, length and height of the container are x, y and z respectively.

a) Show that the surface area S of the container is given by :

$$S = xy + 2xz + 2yz$$

and that

$$z = \frac{10}{xy}$$

(3 marks)

b) Find the dimensions of the container so that a minimum amount of metal is used in its construction. (9 marks)

c) Use an appropriate test to confirm that the minimum condition is satisfied. (8 marks)

Total 20 marks

END OF QUESTIONS

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FORMULA SHEET

Laplace

$$(x) = \int_0^\infty x.e^{-st}\ dt$$

Function	Transform
a	$\frac{a}{s}$
e^{at}	$\frac{1}{s-a}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
sin at	$\frac{a}{s^2+a^2}$
cos at	$\frac{s}{s^2+a^2}$

1st Shift Theorem

$$(e^{-at} . F(t)) = f(s+a)$$

$$(e^{at} . F(t)) = f(s-a)$$

Partial Fractions

$$\frac{F(x)}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$$

$$\frac{F(x)}{(x+a)(x+b)^2} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+b)^2}$$

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Numerical

Euler : $y_1 \approx y_0 + h\, y_0^1$

Improved Euler : $y_1 \approx y_0 + \frac{1}{2} h (y_0^1 + f(x_1, \overline{y}_1))$

Integration by Parts

$$\int u.\frac{dv}{dx} dx = uv - \int v.\frac{du}{dx} dx$$

Fourier

$$F(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x).\cos nx\, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x).\sin nx\, dx$$

Max/Min

C = F (x,y)

Stationary Points $\frac{\partial C}{\partial x} = 0, \quad \frac{\partial C}{\partial y} = 0$

$$\frac{\partial^2 C}{\partial x^2} . \frac{\partial^2 C}{\partial y^2} - \left(\frac{\partial^2 C}{\partial x \partial y} \right)^2$$

Small Changes

$$\delta z \simeq \frac{\partial z}{\partial u}.\delta u + \frac{\partial z}{\partial v}.\delta v + \frac{\partial z}{\partial w}.\delta w$$